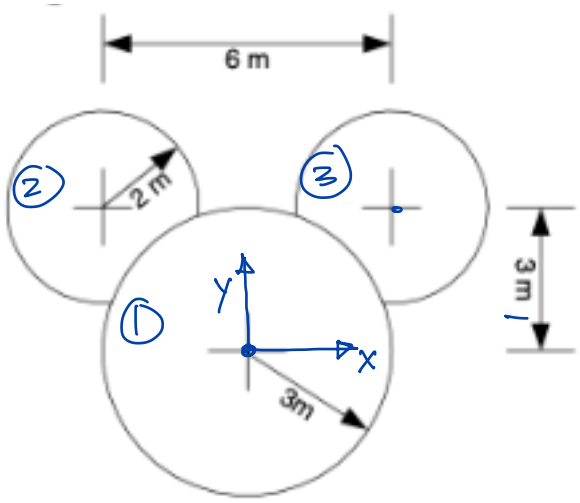
A sign is made from 3 circles of aluminum of thickness 1 cm and density 2.7 g/cm^3. The smaller circles (radius 2 m) are joined to the larger circle (radius 3 m) where they overlap. Find the centre of mass of the sign with respect to the centre of the largest circle. Also find the moment of inertia of the sign about the axis passing through the centre of mass and perpendicular to the plane of the sign.



Find (x_G, y_G) Find I_{COG}

COG

by symmetry, $x_G = 0$

$$y_G = \frac{y_1 m_1 + 2(y_2 m_2)}{m_{TOT}}$$

$$= \frac{\rho t (y_1 A_1 + 2 y_2 A_2)}{\rho t \; A_{TOT}}$$

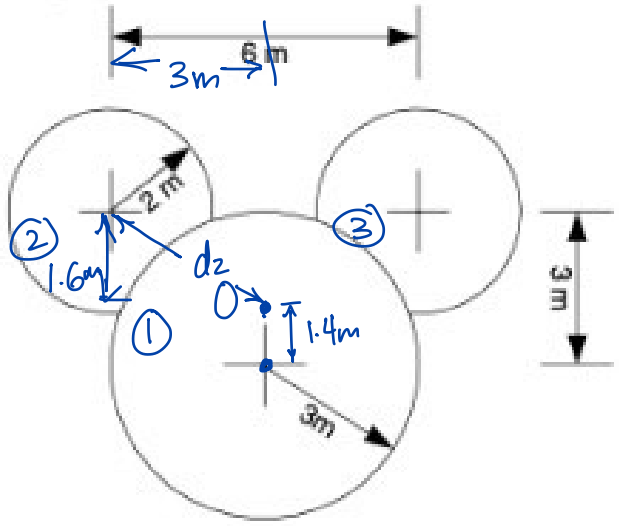
$$= \frac{y_1 (\pi r_1^2) + 2 y_2 (\pi r_2^2)}{(\pi r_1^2 + 2\pi r_2^2)}$$

$$= \frac{0\,(3m)^2 + 2(3m)(2m)^2}{(3m)^2 + 2(2m)^2}$$

$$= \frac{24}{17}\,\frac{m^3}{m^2}$$

$x_G = 0$

$y_G = 1.4m$



MMOI @ COG "O"

① about own COG:

$$I_{zz,1} = \frac{1}{2} m_1 r_1^2$$

about O:

$$I_{zz,1,O} = I_{zz,1} + m_1 d_1^2$$

(d_1 = 1.4m)

$$m_1 = \rho \pi r_1^2 t$$

② about own COG:

$$I_{zz,2} = \frac{1}{2} m_2 r_2^2$$

about O:

$$I_{zz,2,O} = I_{zz,2} + m_2 d_2^2$$

$$d_2^2 = (1.6m)^2 + (3m)^2$$

$$m_2 = \rho \pi r_2^2 t$$

③ $I_{zz,3,O} = I_{zz,2,O}$

MMOI total object @ O:

$$I_{zz,tot} = I_{zz,1,O} + 2(I_{zz,2,O})$$

$$= \frac{1}{2} m_1 r_1^2 + 2\left(\frac{1}{2} m_2 r_2^2\right) + m_1 d_1^2 + 2 m_2 d_2^2$$

$$= \rho \pi t \left[\frac{1}{2} r_1^4 + r_2^4 + r_1^2 d_1^2 + 2 r_2^2 d_2^2\right]$$

$$= \rho \pi t \left[\frac{1}{2}(3m)^4 + (2m)^4 + (3m)^2 (1.4m)^2 + 2(2m)^2 \left[(1.6m)^2 + (3m)^2\right]\right]$$

$$= \rho \pi t (166.62 m^4) = \left(2.70 \frac{g}{cm^3}\right)(3.14)(1cm) \cdot \frac{(100cm)^2}{1m^2} \cdot \frac{1kg}{1000g} \times (166.62 m^4)$$

$$I_{zz,tot\ about\ O} = 14,126\ kg\text{-}m^2$$