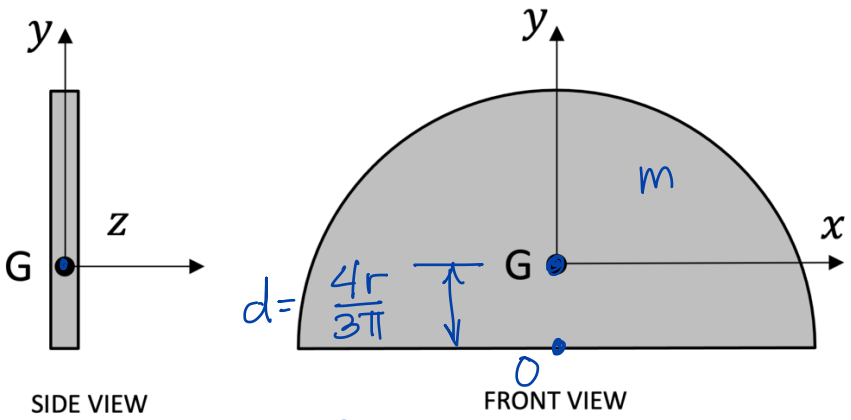
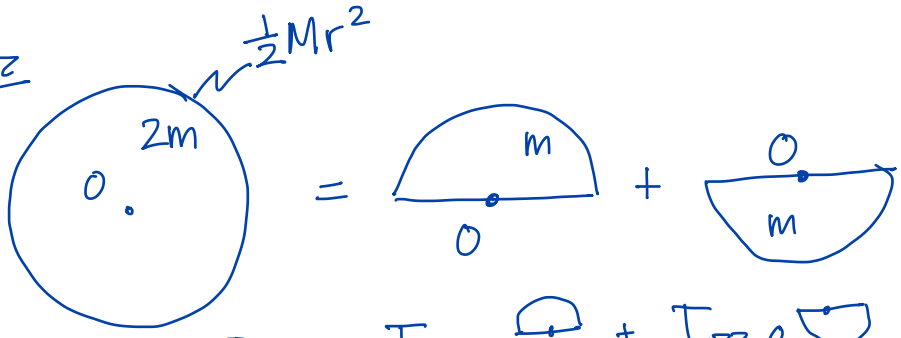
A semicircular thin plate has constant density, a radius of 10 cm, and a mass of 400 g. Find the mass moment of inertia of the plate around the axes (a) x and (b) z passing through the centre of mass.



Find I_{xx}, I_{zz}

Constant density:
COG = centroid

I_{zz}



$$I_{zz,O}\,\odot = I_{zz,O}\,\cap + I_{zz,O}\,\cup = 2\,I_{zz,O}\,\cap$$

$$\Rightarrow I_{zz,O}\,\cap = \frac{1}{2}\,I_{zz,O}\,\odot = \frac{1}{2}\left(\frac{1}{2}Mr^2\right) = \frac{1}{2}\left(\frac{1}{2}(2m)r^2\right)$$

$$= \frac{1}{2}mr^2$$

parallel axes:

$$I_{zz,O}\,\cap = I_{zz,G}\,\cap + md^2$$

(smallest)

$$\Rightarrow I_{zz,G}\,\cap = I_{zz,O}\,\cap - md^2$$

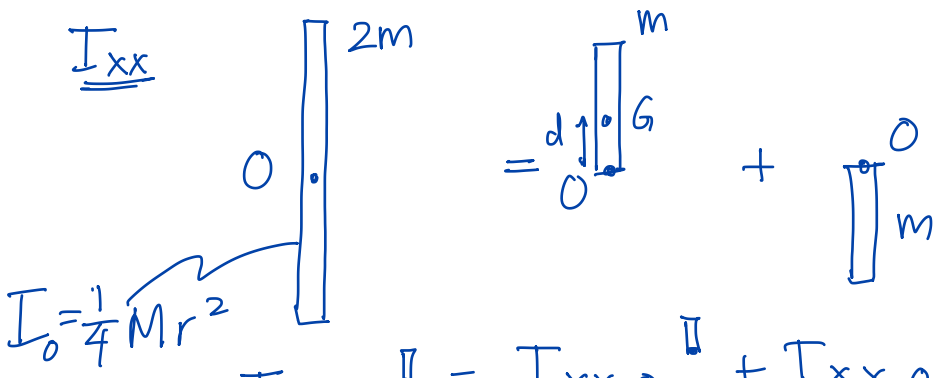
$$= \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

$r = 0.1\,m$, $m = 0.4\,kg$

$$I_{zz,G}\,\cap = 1.28\times10^{-3}\ kg\text{-}m^2$$

circular disk

I_{xx}



$$I_{xx,O}\,\text{‖} = I_{xx,O}\,\text{‖} + I_{xx,O}\,\text{‖} = 2I_{xx,O}\,\text{‖}$$

$$\Rightarrow I_{xx,O}\,\text{‖} = \frac{1}{2} I_{xx,O}\,\text{‖} = \frac{1}{2}\left(\frac{1}{4}Mr^2\right) = \frac{1}{2}\left(\frac{1}{4}(2m)r^2\right)$$

$$= \frac{1}{4}mr^2$$

parallel axes:

$$I_{xx,O}\,\text{‖} = I_{xx,G}\,\text{⊡} + md^2$$

$$\Rightarrow I_{xx,G}\,\text{⊡} = I_{xx,O}\,\text{‖} - md^2$$

$$= \frac{1}{4}mr^2 - m\left(\frac{4r}{3\pi}\right)^2$$

$r = 0.1\,m \quad m = 0.4\,kg$

$$\boxed{I_{xx,G}\,\text{⊡} = 2.79 \times 10^{-4}\,kg\text{-}m^2}$$