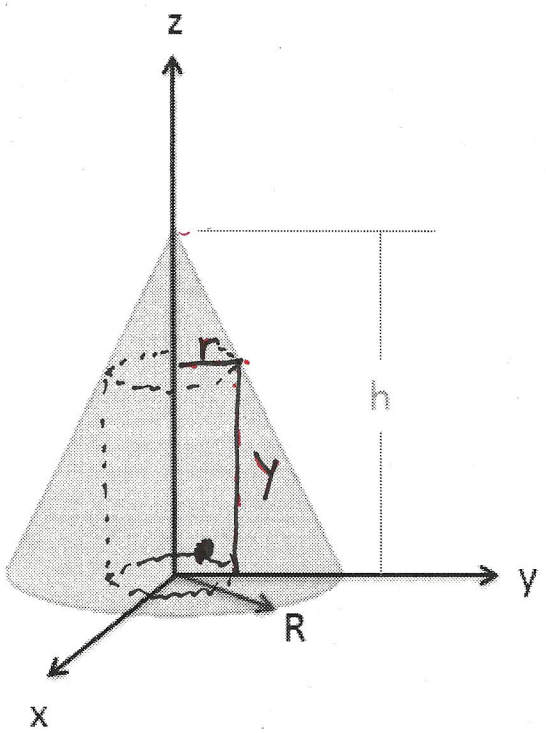
Determine the mass moment of inertia about the z axis for this general cone with base radius R, height h, and mass m.



$$I_{zz} = \rho \int_V r^2 \, dV$$

$$(2\pi r)(y)\, dr$$

$$y = h - \frac{h}{R} r$$

$$I_{zz} = \rho \int_0^R (r^2)(2\pi r)\left(h - \frac{h}{R} r\right) dr$$

$$I_{zz} = \rho 2\pi h \int_0^R r^3 \left(1 - \frac{r}{R}\right) dr$$

$$I_{zz} = \rho 2\pi h \int_0^R \left(r^3 - \frac{r^4}{R}\right) dr$$

$$I_{zz} = \rho 2\pi h \int_0^R \frac{1}{4} r^4 - \frac{1}{5R} r^5$$

$$I_{zz} = \rho 2\pi h \left(\frac{1}{4} R^4 - \frac{1}{5} R^4 \right)$$

$$I_{zz} = \rho 2\pi h \left(\frac{1}{20} R^4 \right)$$

$$I_{zz} = 6 \rho \underbrace{\frac{1}{3}\pi h R^2}_{\text{volume of cone}} \frac{1}{20} R^2$$

mass of cone

$$I_{zz} = (6m)\frac{1}{20} R^2$$

$$\boxed{I_{zz} = \frac{3}{10} m R^2}$$