Problem **20-R-VIB-DY-18**

In this problem, there's a mass attached to a spring. And it's forced by a sinusoidal forcing function. And there's also friction between this sliding mass and the bottom surface. So first, and we're asked to determine the maximum amplitude of the steady state function. So what we need to do first is we need to to, to determine the differential equation of motion. And then from there, determine the steady state solution and determine the maximum of that steady state function. Okay, so first, we start with the free body diagram. And we simply isolate the block, and this block will have the following forces, which I will draw in red. So we have force due to the spring, f k. On the left, because we're assuming x to the right, then we have a force from the forcing function which points to the right, f not sine omega naught t, then we have gravitational force, F G, we have a normal force. And then we have a friction force, F, F. Okay, so these are all the forces and now we do a sum of forces in the x direction and some forces in the y direction. Now some force in the y direction simply yields that. So some forces in the y is equal to zero because there's no acceleration in the y direction, it's just sliding. So we get the following that f g is equal to n. And we get that n is equal to mg. We do some a force, so this is the x direction again. And this is y direction, sum of forces in the x is equal to m a, because we only have acceleration in the x direction, we got the following. f not sine of omega naught t minus f k minus f f equal to x double.so. Again, I replaced acceleration with second derivative of x with respect to time. Now if we rearrange and plug in the values for these times, so fk is mu, m g, it's mu n, and n is mg, f f, and then F k is k x. And we take all of these x turns to the right side, we get the following. f not sine omega naught t is equal to k x or MX double dot plus k x plus mu mg. And again, this new mg comes from the fact that force of friction is mu times the normal force, and we said that the normal force is mg. So now we have this differential equation with x terms on the right side and the T terms on the left side. And we can solve, we can try and solve for the Stacie function function, let's say C function we know is the particular solution. So and to solve for the particular solution, we assume a function of the same form as this. Okay? So we assume that x particular is going to be equal to a constant c times the sine of omega naught t. Okay, so this is the standard way of solving these differential equations are solving specifically for the particular solution. You take a function of the same form as this, but with a different constant C. Now, we have to solve for C. So, what we can do is we can take the derivative of this, well, we can take this and plug it into here as x. So, we can take this and plug it directly into here, but here we have to take the second derivative with respect to time. So, if we take the second derivative of x p Get the following negative c times omega naught squared. So, we get to omega naught out because of the chain rule and then we have sine omega naught t okay so now that we have these two turns, you can take these and plug them into here respectively This doesn't depend on x. So, this is constants that we have and then we're going to have an equation in terms of C and we can solve for that C okay. So, we get the following f not sine omega naught t equals to mass c omega naught squared sine omega naught t plus k times C sine omega naught t plus mu g okay. So, now we can rearrange and we can divide everything by sine omega naught, so, this term goes away this term goes away this term was away and we just have this divided by sine omega naught t okay to simplify and get rid of all those signs and we get the following c times k minus m omega naught squared is equal to f naught minus mu g divided by sine omega naught t okay. So, again I divided everything by omega naught and then I took this term and I took this term and brought it over to this side and then collected the C out of these two terms okay. And now solving for C the constant C we get that c is equal to f naught minus mu g divided by sine omega naught t and this is all divided by k minus m omega naught squared okay. And this is our constancy. case or amplitude is C is this is the amplitude, this part here varies between negative one and one. So, to get the maximum, we just assume that's one and we find C. Okay, so, C here also depends on T. Okay, so what we need to do is, we need to determine when this function here is a maximum. So, it's going to be a maximum when theta sine theta is equal to zero. So, when we plug in zero, we get the following. So, sine theta equals to zero are approximately equal to zero, we get that f naught over k minus omega naught squared. We have all the all of these numbers. So if not we're given is five Newton's divided by k, which we are given is 10 Newtons per meter, minus M, which is five kilograms times omega naught squared, which is three radians per second squared. And so the final answer is C equals to negative 0.143 meters, which is the maximum amplitude