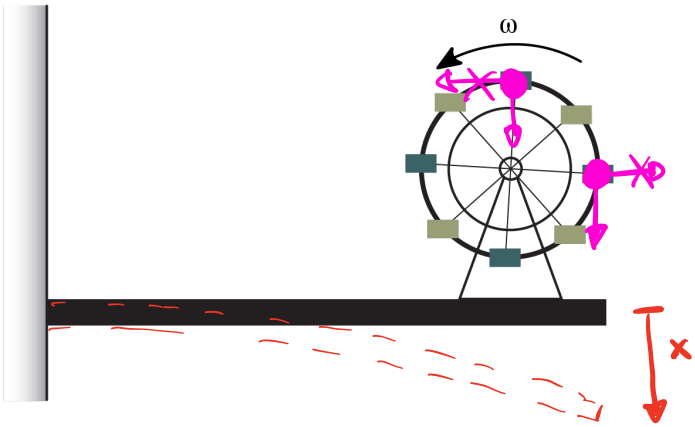
The world's smallest Ferris wheel ($m = 20kg$) is mounted on the end of a horizontal beam for no apparent reason. The wheel is mounted eccentrically in such a way that the eccentricity is equivalent to a mass of $5kg$ located $0.15m$ away from the axis of rotation. The static weight of the Ferris wheel causes a deflection of $20mm$ in the beam. Given that the wheel spins at a rate of 15 rad/sec, find the steady-state amplitude of vibration.



$F_0 = mr\omega^2$

$= (5kg)(0.15m)(15 rad/s)^2$

$F_0 = 168.8N$

$F = KX \quad \Rightarrow \quad K = \frac{F}{\Delta x} = \frac{mg}{\Delta x} = \frac{(20kg)(9.81 m/s^2)}{(0.02m)} = 9810\ N/m$

Steady State:

$$X_p = \underbrace{\frac{F_0/K}{1-\left(\frac{\omega_0}{\omega_n}\right)^2}}_{\text{particular sol only}} \sin \omega_0 t$$

$$X_{max\ ss} = \left| \frac{F_0/K}{1-\left(\frac{\omega_0}{\omega_n}\right)^2} \right| = \left| \frac{(168.8N)/(9810\ N/m)}{1-\left(\frac{15\ rad/s}{22.147\ rad/s}\right)^2} \right| = 0.0318m$$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{9810\ N/m}{20kg}} = 22.147\ rad/s$$

$$X = 0.0318\,m = 3.18\,cm$$