Problem **20-R-VIB-DY-13**

In this problem, there is a juice box, which is attached to a spring and this is attached to a wall or a station, this wheel doesn't turn it stationary, and a force f is applied to this juice box. Okay, and this force f is equal to five sine omega t. so omega being again, five radians per second. And so this is the forcing function, and it's a sinusoidal function. So the first thing we do with all of these problems is we find the freebody diagram. And so in this case, I'm just going to drop the juice box out away from the system, and I'm going to detach it from the spring and from the ground. And I'm going to add all the forces, so we're going to have our force F pulling sinusoidally that way, then we have our force of the spring, which is f k. And then we have, well, just the normal force and the gravitational force, F G. Now from the diagram, we can clearly see that n is going to be equal to f g. And then these two forces added will be equal to the acceleration because we also need to remember that we have an acceleration of the block. Okay, so if we take the sum of forces in the x direction, we get the following minus f k, sorry, let me write that better. f k, is going to be equal to a, and this is a x. Okay? Now, if we write this in terms of what the values actually are, we can plug in F is equal to five sine of five T. And then we have minus f k, which is equal to k x. And then we have m x, which is also equal to m x double dot. Okay, so this is our differential equation for the problem. And if we solve this differential equation, we're going to get a value of x in terms of t, so position as a function of time. And if we rearrange this equation, you can see the following. Okay, if we rearrange and we separate, we pull the x terms on one side, and then we pull all the T terms on the other side, we see that this is a differential equation that we can solve. And there's going to be two parts to it, there's going to be a transient portion and a steady state portion. Okay. And the transient portion will decay to zero as time goes to infinity. But the steady state portion of this equation of the solution of that equation is just going to be constant, and it's going to be a function of this forcing frequency. So the force F has an amplitude of five Newtons with an Omega of five radians per second. Okay, um, so this is what we call here, omega not forcing frequency, and this is what we call F, not the magnitude of that forcing frequency. And of that force, sorry. And so I'm not going to solve this differential equation. But you know that the natural frequency is determined by the left here, equating this term here to zero, because the natural frequency doesn't depend on the forcing function. So and then we can solve for rates as usual, by eliminating taking this m term and dividing everything by m. so that nothing is in front of this x double dot term. And whatever is in front of the x term, we take the square root and that's the natural frequency. So we have omega n being equal to the square root of k over m. And so in this case, we have square root of 25 on Newton's per meter over M, which is two kilograms. And so omega n is just going to be equal to the square root of 25 over two radians per second. Okay. Now we also so that's how we get the natural frequency. Now we want to find that maximum amplitude. So the maximum amplitude is going to be determined by the amplitude of that particular solution, hey, because that is the steady state part of the solution. And so we know that when we have something of this form x p, so the position x or the amplitude is going to be given by f naught over k divided by one minus omega not over omega n, all squared sine of omega naught t. Okay, so, this is the particular part of the solution. And again, it's going to be a sinusoid. That depends on the frequency the forcing frequency omega naught. But this amplitude here also depends on omega n, the amplitude of the forcing function and K. Okay. And so this is the form of that solution if we have a sine term here. And if we want to find the maximum amplitude, we know that this term here, the sine term goes from zero or negative one to one. So the maximum this can be is going to be one, so it's just going to be this value over here. And we if we plug in all of the knowns, and the problem, we can actually solve for this because we just found the natural frequency, we're given omega naught, we're given k and this is the maximum amplitude of 5k. So this term here is the maximum amplitude x max at steady state, specifically cc because they chose the particular solution. Okay, so x max is going to be equal to f naught, which we said is five Newton's over k, which is 25 Newton's per meter, and I'll put that in brackets. And this is all divided by one minus omega naught, which we said was five radians per second, divided by omega n, which is square root of me make that bigger, heard of 25 over two. And, sorry, and that's still radians per second. And that's all squared. Okay. And if we actually solve for this, we get negative zero point or not negative, yet 0.2 meters, that's going to be equal to x max. And that is the first part of the solution, the maximum amplitude. Next, we're asked to find the magnification factor. So magnification factor, and f is going to be equal to the following form one over one minus omega naught not over omega n squared. And this is going to be equal to when we plug everything in, we get one over one minus omega naught, which is five radians per second divided by omega n squared, which is square root of 25 over two radians per second, all squared, and when we get when we solve for this, we are going to get negative one. And again, this is a unitless parameter so there's no units to the magnification factor. And this is the second part of our solution.