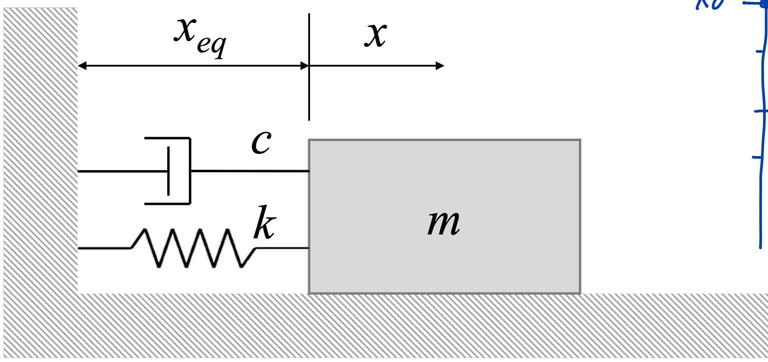
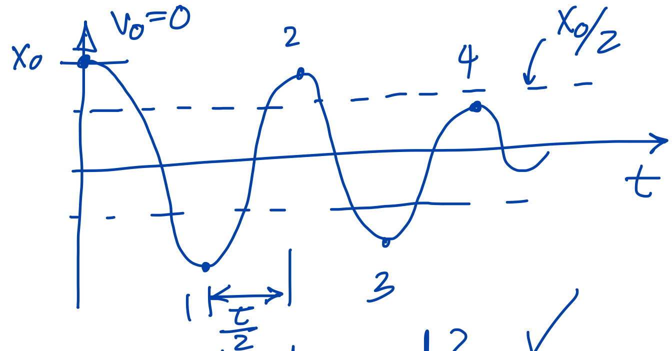
A 20 kg block on a frictionless surface is attached to a spring (k = 700 N/m) and a damper (c = 35 N-s/m). If the initial perturbation, x_0, is 0.2 m ($v_0 = 0$), how many half-cycles will it take for the amplitude of the oscillation to peak at half the original displacement or less (i.e. $|x_{peak}| <= |x_0/2|$)?





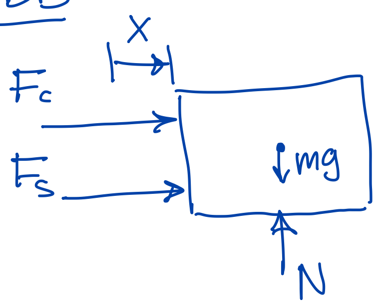
underdamped? ✓

$c^2 < 4mk$

$(35\,\frac{N\cdot s}{m})^2 < 4(20\,kg)(700\,N/m)$

$1225 < 56000$

FBD



$F_s = -kx$

$F_c = -c\dot{x}$

$\Sigma F_x: \; F_c + F_s = ma_x = m\ddot{x}$

$\Rightarrow -c\dot{x} - kx = m\ddot{x}$

$\Rightarrow \underbrace{m\ddot{x} + c\dot{x} + kx}_{\text{must all be positive}} = 0$

$\omega_n = \sqrt{\frac{k}{m}} = 5.92\,rad/s$

$\zeta = \frac{c}{2m\omega_n} = 0.148$ (no unit)

$\Rightarrow \; \ddot{x} + \underbrace{\frac{c}{m}}_{2\omega_n\zeta}\dot{x} + \underbrace{\frac{k}{m}}_{\omega_n^2}x = 0$

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 5.85\,\frac{rad}{s}$

solution of the form: $x(t) = [c_1 \sin\omega_d t + c_2\cos\omega_d t]\, e^{-\omega_n\zeta t}$

$c_1 = \frac{\cancel{\dot{x}_0}^{\,0} + \omega_n\zeta x_0}{\omega_d}$ $\quad c_2 = x_0$

consider peaks: $\frac{\tau}{2} = \frac{\pi}{\omega_d}$ or $t_n = \frac{n\pi}{\omega_d}$ where $n = 1, 2, 3 \ldots$ (integers)

$$x(t_n) = \left[\frac{\omega_n \zeta x_0}{\omega_d} \sin\left(\cancel{\omega_d} \cdot \frac{n\pi}{\cancel{\omega_d}} \right) + x_0 \cos\left(\cancel{\omega_d} \cdot \frac{n\pi}{\cancel{\omega_d}} \right) \right] e^{-\omega_n \zeta \frac{n\pi}{\omega_d}}$$

$$\sin(n\pi) = 0 \qquad \cos(n\pi) = \pm 1$$

$$x(t_n) = \left[0 + x_0(1) \right] e^{-\omega_n \zeta \frac{n\pi}{\omega_d}}$$

Find:

$$|x(t_n)| \leq \left| \frac{x_0}{2} \right|$$

$$\left| \cancel{x_0} e^{-\omega_n \zeta \frac{n\pi}{\omega_d}} \right| \leq \left| \frac{\cancel{x_0}}{2} \right|$$

$$\left| -\frac{\omega_n \zeta n\pi}{\omega_d} \right| \leq \left| \ln\left(\frac{1}{2}\right) \right|$$

$$|-n| \leq \left| \frac{\omega_d}{\omega_n \zeta \pi} \cdot \ln\left(\frac{1}{2}\right) \right|$$

$$\leq \left| \frac{5.85 \text{ rad/s}}{5.92 \text{ rad/s} (0.148)(3.14)} \cdot (-0.693) \right|$$

$$|-n| \leq |-1.474|$$

$$n \geq 1.474$$

$$\boxed{n = 2}$$