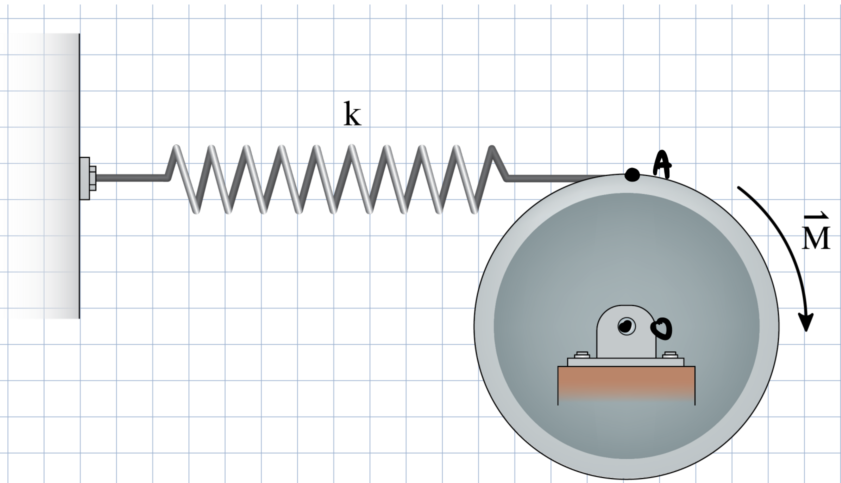
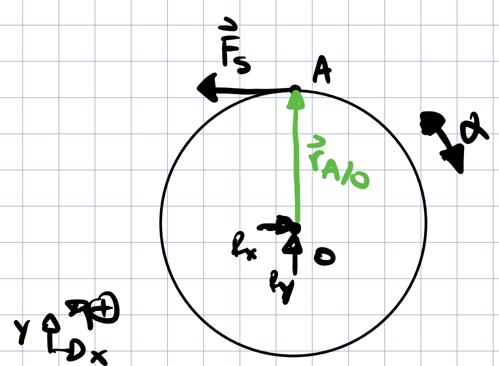
A spring with spring constant $k = 90$ N/m is connected to a disk with a mass 15 kg and radius $r = 1$ m. A moment of $\vec{M} = -40\hat{k}$ N · m is required to keep the disk in static equilibrium. At time $t = t_0$ the moment is suddenly removed, and the disk begins to experience oscillatory motion due to the spring. With what period, τ, does the system oscillate? (You may assume $\sin\theta = \theta$)



$I_o = \frac{1}{2} m r^2 = \frac{1}{2}(15\,kg)(1\,m)^2$

$I_o = 7.5\ kg\,m^2$

FBD:



$\vec{F}_s = -k\vec{x} = -kr\theta$

$\to \vec{\alpha} = \ddot{\theta}$

$\sum M_o = I_o \alpha \quad \to \quad \vec{r}_{A/O} \times \vec{F}_s = I_o \vec{\alpha}$

$\to r(-kr\theta)\hat{k} = I_o \ddot{\theta}$

$I_o \ddot{\theta} + k r^2 \theta = 0$

$\ddot{\theta} + \omega_n^2 \theta = 0$

$$\ddot{\theta} + \frac{Kr^2}{I_o}\theta = 0$$

$$w_n = \frac{2\pi}{\tau}$$

$$w_n^2 = \frac{Kr^2}{I_o} = \left(\frac{2\pi}{\tau}\right)^2$$

$$\tau = 2\pi\sqrt{\frac{I_o}{Kr^2}} = \boxed{1.81 \text{ sec} = \tau}$$