Problem **20-R-VIB-DY-50**

In this problem, a triangle is made of three bars. And each bar is three meters in length and two kilograms in mass and oscillates about one of the corners, we're asked to determine the natural frequency of this system. So the first thing we do, when we need to find the natural frequency the system is we need to draw the system in a displaced position. And based on that, we can then do the freebody diagram and write in all the forces and then do either a sum of forces or some moments, depending on the situation. Okay, so the displace system in this case, would be the triangle pointing in some kind of direction like this. So in this case, I'm going to draw the freebody diagram down here, this is going to be the vertical direction. And the triangle is going to be slanted to the side like this. Right. So again, this is not a perfect equilateral triangle, but in real life, it would be a perfect equilateral triangle. And the forces are the force due to gravity. So f g, on this side here is pinned. Okay. So what we're going to do in this case, is take a sum of moments about Oh, over here, okay. Because in this case, everything is rotating. So that's why we're taking some moments. And so the equation is going to be a sum of moments. And we're going to take about Oh, because there are reaction forces at over here. So we have a driving green, our y, and our x, we do have reaction forces, if we take the moment about oh, well, that will be that will cancel those two reaction versus this is going to be equal to i, not alpha. Okay. And so what we need to do here is, first of all, take the sum of moments, and then figure out what AI is. Okay, so what I know is, also we can actually start with AI, not because it's it's simpler. Um, so I not is the moment of inertia about Oh, here. So this point here, and there's gonna be three components to it. So two of them are basically going to be these two bars, and again, starting from the end of the bar, and we just multiplied by two, because these two bars are identical in size and weight. And then we have a third bar over here, which we can take, I approach to the center here and then use parallel axis and translate this distance over here. Okay. So that's how we're going to calculate I about oh, I O is going to be equal to the two bars, so times two, times 1/3 M L squared. And then we have the bottom bar, which has two terms to it. So I'm going to put it in square brackets 112 M L squared plus MD squared. Okay. So this is equal to two times 1/3 L squared plus 112. L squared plus m and this distance d here is going to be the length of bar times sine of 60 degrees. So l times sine of 60 degrees. Now, why is that? It's because this angle here is 60 degree degrees, this is also 60. And this is also 60 degrees, because it's an equal lateral triangle. And then since we're trying to find this distance here, so this height over here, so from here to here, this is d, we just take the sine of 60 times this hypotenuse, which is three meters, and that's going to give us d over here. Okay? And so that's why we have l sine 60. And we can we have all of these parameters. We can plug everything Again, so we have two times 1/3 times the mass, which is two kilograms, sorry, times two kilograms times L, which is three meters squared plus 112 times two kilograms times three meters squared plus two kilograms, times three meters. And sine is 60 degrees. Therefore we get a final answer of if not, is equal to 27 kilograms, meters squared. Okay. So now we have I not, we just need to find, we just need to solve the left side of the moment equation. So the sum of moments and as you can see about Oh, there's only one force that creates a moment, and this force is going to have a moment arm that is this is the moment arm because the force points directly downwards, we need to find that distance are over there. Okay. So the sum of moments where Oh, is going to be equal to r times f g, which is equal to r times G. Okay? Now how do we find that distance r? Well, we need to find the location of the center of gravity, because that's where the force of gravity acts. And since these two sides are equal, it's going to be along this vertical line. Okay? So that's where along the Y plane, but the x coordinate, we're not sure. But since we know it's a triangle, we know that for a triangle on the center of gravity is two thirds of the way down. Okay? So this is actually where g is located. Okay. So that distance there, we're going to call H. Okay, so this distance from this point here to G, we're going to call distance h, okay. and if we calculate, if we go on the sign to triangle here, h is this distance that is slanted like that. Okay, so this distance from here to here, that's H. Okay. And, to find our, what we need to do is we need to use theta, okay? Which theta is the angle that we're shifting everything back. Alright. So we can start plugging things in over here. So we have our, which is now going to be in terms of H. And in terms of theta. So if you to find this distance over here, what we can do is we can simply take h times sine of theta, theta being the angle that we displace by over here. Okay, so I'm going to draw a bigger diagram of the, that small triangle that I draw Drew, and so we have the center of gravity over here, oh, over here, we have G over here, and then we have the normal over here, okay. So theta is going to be in let me draw everything in the correct colors. Theta is going to be this angle over here, the angle by which we are displacing the system, okay? And the distance R is going to be this instance over here are, okay. So we already said that h is going to be two thirds of the way down of that height. So H is going to be equal to two thirds of D. And we said that D was l sine theta. So we have h being equal to two thirds times L times sine of 60 degrees. Okay, so now that we've displaced the system, we know what ages because age is constant with this displacement theta, which is a new discipline. And we're adding on, we can find R, which is the moment arm for that force F g that acts at G over here. Okay, so now we can replace r with the following h. So we have h sine theta, m g, which is also equal, we can plug in this equation for h two thirds l sine 62 thirds l sine and 60 degrees, sine theta, m g. Okay. And now we can plug everything into the momentum equation over here. And so and we have a differential equation, and from the differential equation, we can determine theta, or we can determine omega n. Okay, so on the left side, we have two thirds L, sine of 60 degrees, sine of theta, and G. And on the right side, we have I not alpha. Okay, so and this is going to be sorry, there's a negative sign in front here. So this is a differential equation. Well, it is a differential equation because this is equal to negative, not theta double dot, but we have sine theta here, not theta. Okay. So this is complex, we can't directly solve, but what we can use, we can use small angle approximation. So if theta is small, then we can use the small angle approximation that says that sine theta is approximately equal to theta. so we can actually replace that term with data and get the following. Two thirds l sine 60 degrees and G theta plus by not theta double.is equal to zero. And we know that once we have an equation in this form, to find the natural frequency, we just need to get rid of whichever term is in front of the theta double dot term and make it one and then whichever term is in the front of the theta term, or the just the variable without a derivative is the natural frequencies squared, okay. So the natural frequency omega n will be equal to two thirds, l sine of 60 degrees and G over I not square rooted. Okay. And if we plug all the values in, we got two thirds L, which is three meters times sine of 60 degrees, times n, which is two kilograms, the mass times 9.81 meters per second squared divided by 27 kilograms per meters, kilograms meters squared. Take the square root of it, and you get the following value, again, is equal to 1.9 radians per second. And this is the final answer.