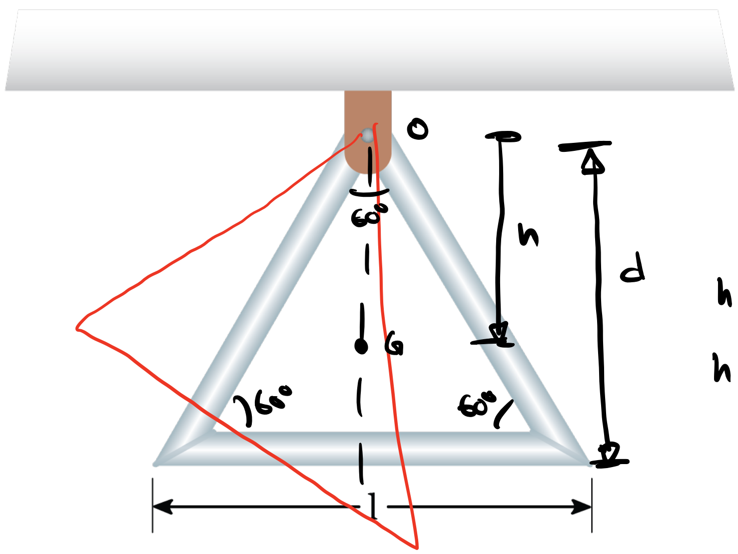
An equilateral triangle is made up of three individual bars. Each bar is 3m in length and 2kg in mass. Given that the frame is hung on the ceiling by one of its corners, determine the natural frequency.

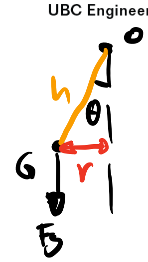


$$h = \frac{2}{3} d$$

$$h = \frac{2}{3} \ell \sin 60°$$

FBD

$$\Sigma M_o = -I_o \alpha$$



$$I_o = 2\left(\frac{1}{3} m\ell^2\right) + \left[\frac{1}{12} m\ell^2 + md^2\right]$$

$$= 2\left(\frac{1}{3} m\ell^2\right) + \frac{1}{12} m\ell^2 + m(\ell \sin 60°)^2$$

$$= 2\left(\frac{1}{3}(2kg)(3m)^2\right) + \frac{1}{12}(2kg)(3m)^2 + (2kg)((3m)\sin 60°)^2$$

$$I_o = 27 \text{ kgm}^2$$

$$\Sigma M_o = r F_g = r\, 3mg = h \sin\theta\, 3mg = \frac{2}{3} \ell \sin 60° \sin\theta (3) mg$$

$$\frac{2}{3}\, l \sin 60^\circ \sin\theta\, (3) mg = -I_o \alpha = -I_o \ddot{\theta}$$

If θ is small $\Rightarrow$ Small angle approximation $\Rightarrow$ $\sin\theta \approx \theta$

$$2 l \sin 60^\circ mg\, \theta + I_o \ddot{\theta} = 0$$

$$\omega_n = \sqrt{\frac{2 l \sin 60^\circ mg}{I_o}} = \sqrt{\frac{2\,(3\,m) \sin 60^\circ\, (2\,kg)\,(9.81\,m/s^2)}{27\,kg\,m^2}}$$

$$\boxed{\omega_n = 1.9\ rad/s}$$