Problem **20-R-VIB-DY-49**

In this problem, a mass is attached to a spring. And this mass has an initial velocity. And we're asked to find what is the max amplitude, and what is the actual amplitude at time equals to 10 seconds. So the first thing we need to do is we need to draw a freebody diagram and determine all of the forces that are acting on the base mass. So I'm going to draw the mass as a rectangle. And this is going to include both the mass of the plate and also the target, and also the mass of the arrow. So this is going to be total. And I added both masses, because the question is specifically states that the target sticks, the arrow sticks to the target, so the masses are added. And we have a spring force. So since our initial velocity is in that direction, there, we are going to have a spring force that is opposite to the displacement, because a compression leads to force opposite to the direction of compression. So the compression for the compression of the spring leads to Fs towards the right. Okay. And again, when we do a sum of forces, this will equal to the mass times the acceleration. Okay? So, again, the block will have an acceleration. The, these are all the forces, there's no friction, there's nothing else. So there's only one force acting on this block. Okay. So let's do it some forces. So this is this is a 1d problem. So the extraction is positive in that direction. So a sum of forces in the extraction equals to M a x. And in this case, we're going to call e x a, because there is no other acceleration. There's no y acceleration in this one D. Okay, so we can now plug in the force in spring. So we have Fs equals to a x, okay, which is equal to us. Alright. S is equal to m, a, and this is total. Okay. And we can actually plug in what Fs is. So Fs is equal to k x. Okay, because this is just the spring constant times the displacement, and we end up with the following equation are actually negative dx. So we end up with negative k x is equal to a total and total a. And if we rearrange, we got the following differential equation total a plus k x is equal to zero. And this is a differential equation because A is just the the second derivative of x with respect to time. And so this equation becomes total x double dot plus k x is equal to zero. And this is a differential equation then we can solve. So what we do to solve this differential equation is we divide everything by m to have this term have a one in front, not an M total. And so when we rearrange, we get the following. x double dot plus k over total, x is equal to zero. And we know that this equation has a solution of the form x is equal to A sine omega t plus B goes on Vega and ti, were this term here is equal to omega n. Okay, or omega n squared actually. So we can solve this. And we can plug in this omega n squared into here and here, but we still don't know the coefficients a and b, and those are determined by the initial conditions. Okay, so the initial conditions are the following x zero, this is a time equals to zero, you're going to be equal to zero meters, and x.of time equals to zero is equal to V naught, which is equal to are given five meters per second. Okay, so this is how we determine these coefficients a and b. Okay, so let's plug this term. Let's plug in t equals zero into this equation, and we can see that the first term cancels out because sine of zero is zero, cosine of zero is one, so we have x equals two B time equals to zero. So that will will mean that B has to be zero because everything has to be equal to zero. Okay, so, x of zero is equal to a times zero, plus b times one, which is equal to b. And since this has to be equal to zero, therefore, b is equal to zero. Okay, so that is the first simplification, then we have the second initial condition, so this is one, we have a number two, and this is x.of, zero is equal to five. Okay, now, let's find x.so x.is just the derivative with respect to time of this equation over here. So, if we take the derivative of the first time we get a omega n coas omega n t, and this omega n comes out due to the chain rule, and then we have we do the same to the second term, but instead of assigning we have a cosine, so we get b omega n times sine omega n t, and sorry, this should be a negative because the derivative of cosine is negative sine, okay, but we can already scrap this out because b is zero. So, that term is already canceled. So, we have that V naught is equal to A omega n times one, because cosine of zero is one node plugged into u goes to zero. So, this means that a is equal to v not over omega n, okay, equal to five meters per second over omega n. So, now, we have both coefficients, and we get the following equation x of t is equal to and we just have that first term a sine omega n t, where A is B naught over omega n v naught over omega n times sine of omega n t, where omega n is equal to the square root of k over m total, okay. And we can actually calculate what this value is based on the given parameters on so k being v 50 Newtons per meter and the total bass being six kilograms on because we have five kilograms plus one. So, that will be equal to 50 Newtons per meter over six kilograms is equal to 2.887 radians per second. Okay. And this will we can plug this directly into here and we will get we can solve for anything tomorrow. Any products position in terms of time. Now, first, we're asked to find the max amplitude. So the max amplitude is the max, maximum location at which this equation is equal to. So in this case, we can see that sign here oscillates between negative one and one. So the maximum for this portion of the equation is just one, okay? And this is a constant, so the maximum is just this constant times one. So it's essentially V naught over omega n, okay. So, Max A is equal to V naught over omega n, okay. And we can actually plug in these values for V naught is five meters per second, and omega n is 2.887 radians per second, and we get that the max amplitude is 1.73 meters. Okay, so this is the first part of the solution. And then we can solve for a specific amplitude at a given time, which is 10 seconds, okay, so, at t equals to 10 seconds, x is going to be equal to P naught over omega n and sine of omega n times 10 seconds, okay. And this is going to be equal to again five meters per second divided by 2.887 radians per second times sine of 2.887 radians per second times 10 seconds, and X of t equals to 10 seconds, it's going to be equal to negative 1.44 meters. And this is the second part of the solution where we found the amplitude at 10 seconds.