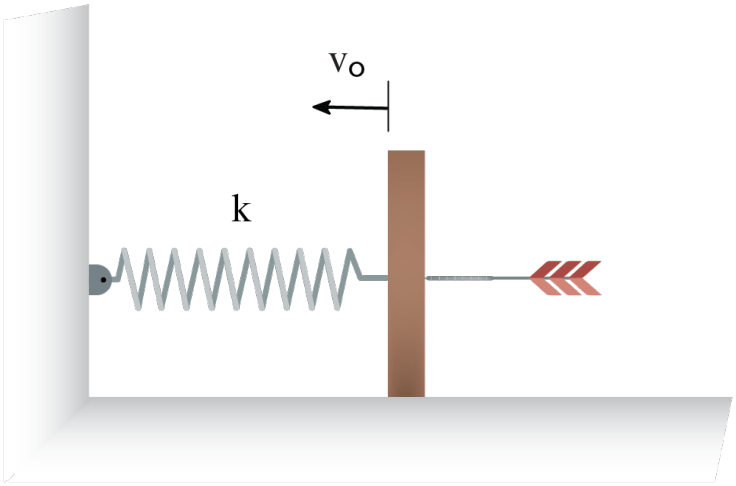
A target, with $m = 5kg$, in an archery range is attached to a wall with a spring, $k = 50\ N/m$. An archer shoots a $m = 1kg$ arrow which sticks into the target. As a result of an impact with the arrow, the target has a horizontal initial velocity $v_o = 5m/s$. Determine the maximum amplitude and the displacement at $t = 10s$.



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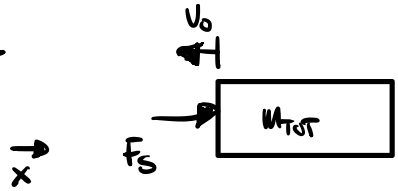

$\sum F_x = m a_x \qquad F_s = m_{TOT} a_x \Rightarrow F_s = m_{TOT} a$

$F_s = -Kx$

$-Kx = m_{TOT} a \Rightarrow m_{TOT} a + Kx = 0 \Rightarrow m_{TOT} \ddot{x} + Kx = 0$

$\text{LD} \quad \ddot{x} + \underbrace{\frac{K}{m_{TOT}}}_{\omega_n^2} x = 0 \Rightarrow x = A \sin \omega_n t + B \cos \omega_n t$

$\text{IC's} \Rightarrow x(t=0) = 0m \qquad \dot{x}(t=0) = V_0 = 5m/s$

$\text{①} \quad x(0) = A(0) + B(1) = B = 0 \Rightarrow \underline{B = 0}$

(2) $\dot{x}(0) = 5 \Rightarrow \dot{x} = A\omega_n \cos\omega_n t - B\omega_n \sin\omega_n t$ (B → 0)

$$V_0 = A\omega_n (1) \Rightarrow A = \frac{V_0}{\omega_n} = \frac{5\,m/s}{\omega_n}$$

$$x(t) = \frac{V_0}{\omega_n} \sin\omega_n t \qquad \text{where} \quad \omega_n = \sqrt{\frac{k}{m}}$$

$$\omega_n = \sqrt{\frac{50\,N/m}{6\,kg}} = 2.887\,\frac{rad}{s}$$

Max Amplitude: $MAX\,A = \frac{V_0}{\omega_n} = \frac{5\,m/s}{2.887\,rad/s} = \boxed{1.73\,m}$

At $t = 10\,sec$ $\quad x = \frac{V_0}{\omega_n} \sin\omega_n(10sec) = \frac{5\,m/s}{2.887\,rad/s} \sin\left(2.887\,\frac{rad}{s}(10s)\right)$

$$\boxed{x(t=10\,sec) = -1.44\,m}$$