15-1 WP001

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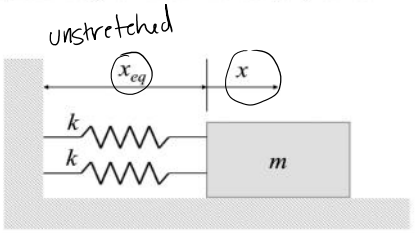
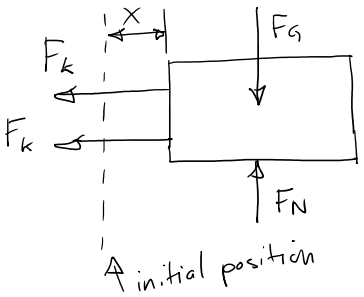
Question 1:

Find an expression for the angular natural frequency of the following system, and find the maximum amplitude of vibration of the system with mass m = 10 kg and spring constant k = 200 N/m when given an inital displacement of x_0 = 0.1 m and an initial velocity of v_0 = 0.3 m/s.



FBD perturbed position



$\Sigma F_x: -F_k - F_k = m a_x = m\ddot{x}$

$F_k = kx$

$\Rightarrow -2kx = m\ddot{x}$

$\Rightarrow m\ddot{x} + 2kx = 0$ ← all terms positive → good!

normal form:

$\ddot{x} + \frac{2k}{m} x = 0$ $\quad \omega_n^2 = \frac{2k}{m}$

$\ddot{x} + \omega_n^2 x = 0$

$\boxed{\omega_n = \sqrt{\frac{2k}{m}}}$ *

$k_{eq} = 2k$

$\Rightarrow \omega_n = \sqrt{\frac{k_{eq}}{m}}$

ODE solution: $x(t) = C \sin(\omega_n t + \phi)$

C ~ amplitude

$$C = \left(\frac{V_0^2}{\omega_n^2} + X_0^2\right)^{1/2} = \left(\frac{V_0^2 m}{2k} + X_0^2\right)^{1/2}$$

$$= \left(\frac{(0.3\,m/s)^2 (10\,kg)}{2(200\,N/m)} + (0.1m)^2\right)^{1/2}$$

$\boxed{C = 0.1107\,m} > X_0$